



Steel-concrete composite beams Part b

**(ref. TGC 10 chap.4 -5
and TGC 11 § 10.5)**

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RESSLab**



Overall content

- Introduction (recall)
- Section strength
- Structural analysis
- Structural verification, ULS (ELU)
- Serviceability check, SLS (ELS)
- Connection

Contents part b

- Structural analysis
- Structural verification, ULS
 - Construction stage and loads
- Serviceability check, SLS
- Special loads to consider
- Appendices

Structural analysis

TGC 10 § 4.7.4

TGC 11 §10.5.3

Principle for determining internal forces, and calculation method

Position 1, max moment in span
Position 1, moment max en travée

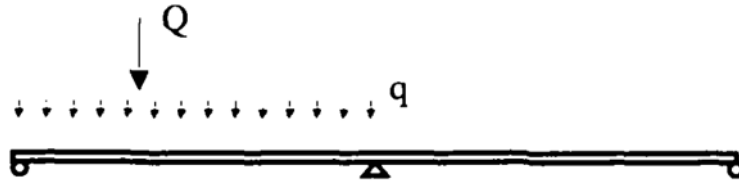
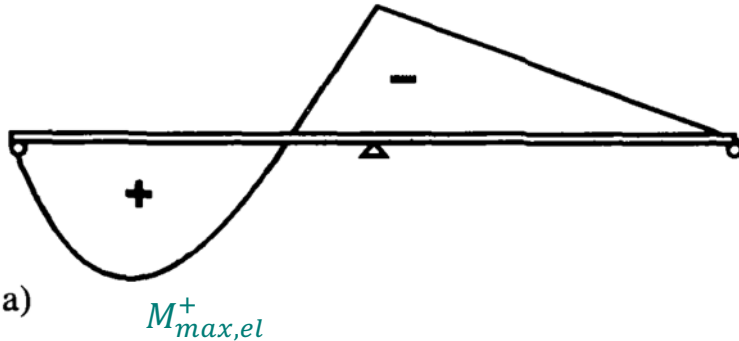
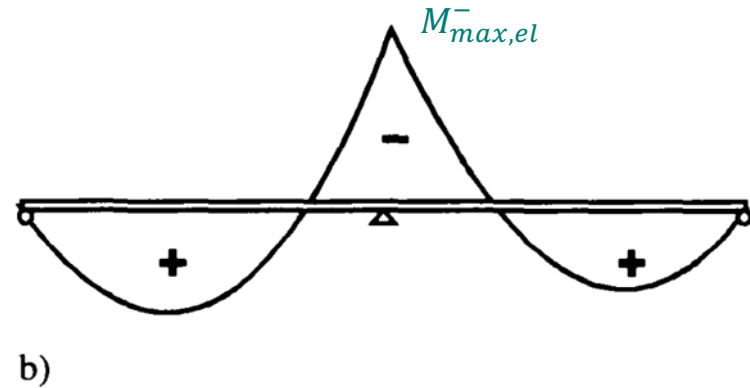
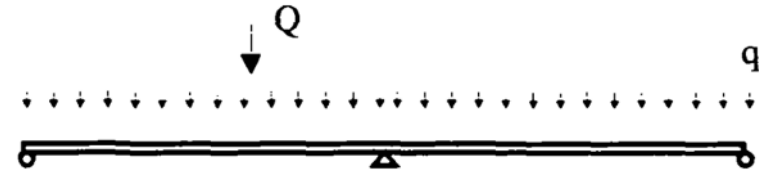


Diagramme des moments Moment diagram

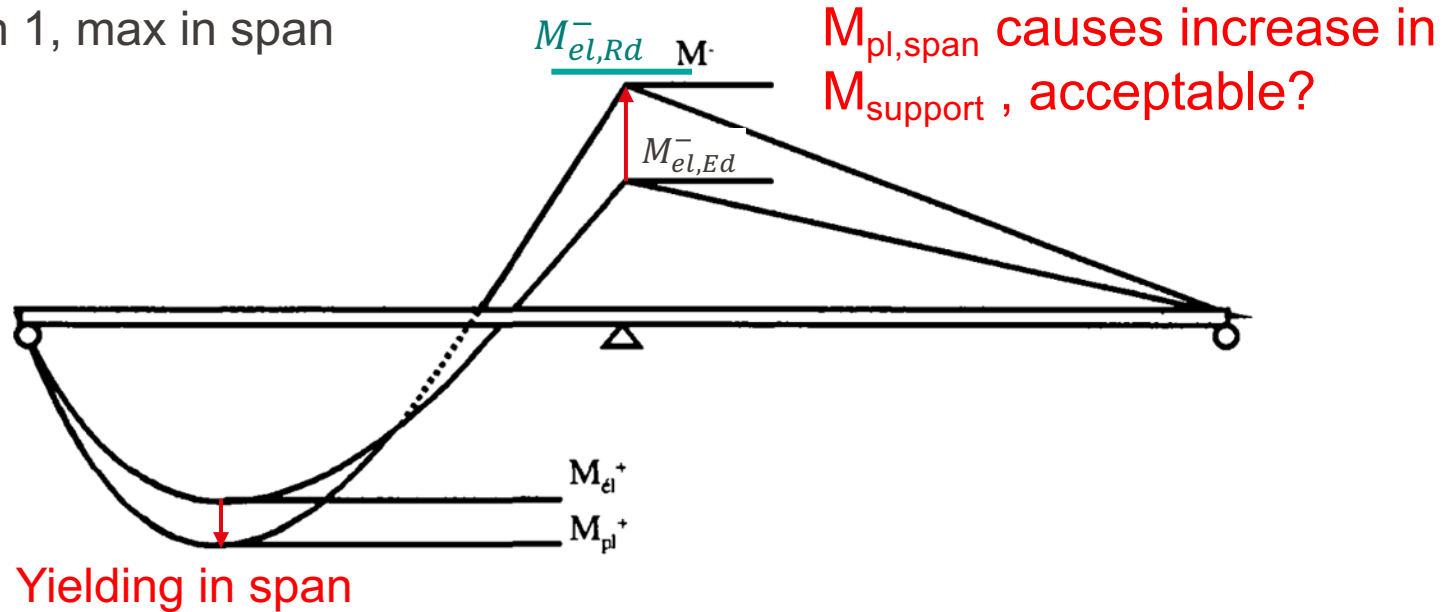


Position 2, max moment at support
Position 2, moment max sur appui



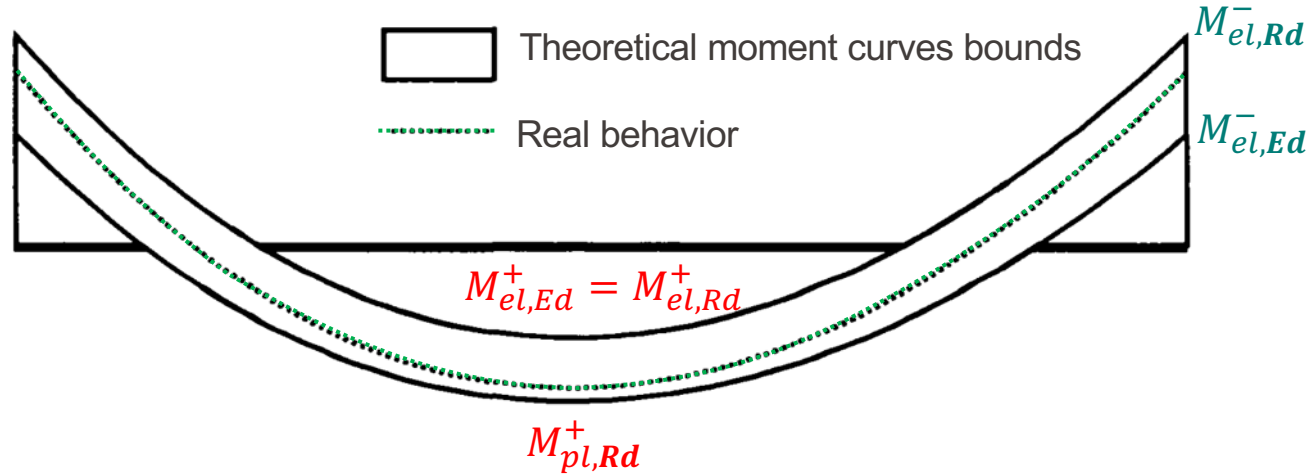
Principle for determining internal forces, and calculation method

Position 1, max in span



Principle for determining internal forces, and calculation method

Position 1, max in span

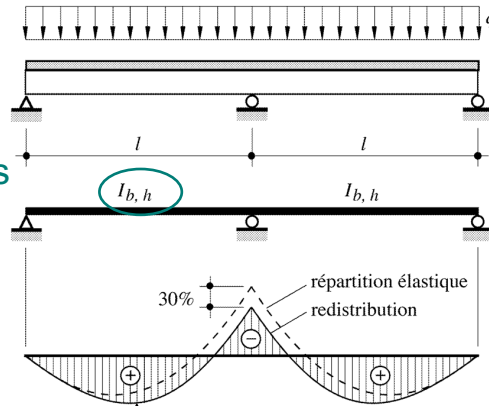


Determining internal forces, M_{Ed} : Elastic calculation, what inertia(ies)?

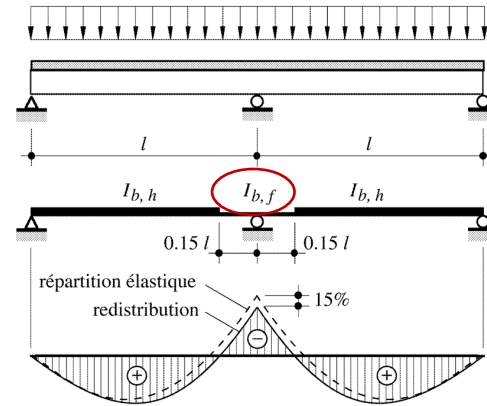
1st approximation take $I_{b,h} = \text{cste}$ (span, $b_{\text{eff}} = \text{cste}$) and $L_{\text{span}} = \text{cste}$

Influence of variable stiffness, simplified method (TGC 11, fig.10.25, Tab. 10.26)

Homogeneous cross-section (span) inertia



$M_{tr,final} \cong M_{tr} + 30\%M_{ap}$ a) Constant inertia



Inertia of cracked cross-section (support)

b) Variable inertia

Section class in zones M-

	Calcul élastique des efforts intérieurs avec:	Section de classe 1	Section de classe 2
a)	• inertie constante (section non fissurée), fig. 10.25(a)	40%	30%
b)	• inertie variable (section fissurée sur appui), fig. 10.25(b)	25%	15%

Max. % redistribution
SIA 264 § 4.2.3.4

SIA 264, redistribution des moments obtenus par calcul élastique (english next)

4.2.3.4 La redistribution des moments fléchissants obtenus par une analyse élastique peut être modifiée, tout en respectant l'équilibre, pour tenir compte des effets de la fissuration du béton, du comportement non élastique des matériaux et du voilement local des éléments structuraux en acier.

Pour des poutres mixtes continues, de hauteur constante, il est possible de modifier les moments fléchissants résultant d'une analyse élastique en réduisant les moments fléchissants maximaux sur appuis selon les pourcentages indiqués dans le tableau 4 et en augmentant en conséquence les moments en travée correspondants.

Tableau 4: Réduction maximale des moments fléchissants résultant d'une analyse élastique

Classe de section dans les zones de moments fléchissants négatifs	1	2	3	4
Calcul élastique sans prise en compte de la fissuration	40%	30%	20%	10%
Calcul élastique avec prise en compte de la fissuration	25%	15%	10%	0%

4.2.3.5 La résistance ultime plastique des sections en travée des poutres continues et des traverses de cadres tenus latéralement qui remplissent les conditions de la classe de section 2 peut être utilisée lors de la vérification, même si les sections ne remplissent pas ces mêmes conditions dans la zone des appuis et pour autant que la résistance élastique de la section dans la zone des appuis ne soit utilisée qu'à 90% au plus, pour le cas de charge considéré. La vérification au déversement dans la zone des appuis doit alors être effectuée selon la norme SIA 263.

SIA 264, redistribution of moments obtained by elastic calculation

4.2.3.4 The redistribution of bending moments obtained by elastic analysis can be modified, while maintaining equilibrium, to take account of the effects of cracking in concrete, inelastic behaviour of materials and local buckling of structural steel members.

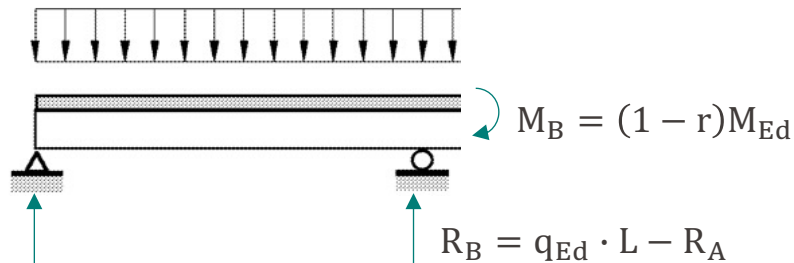
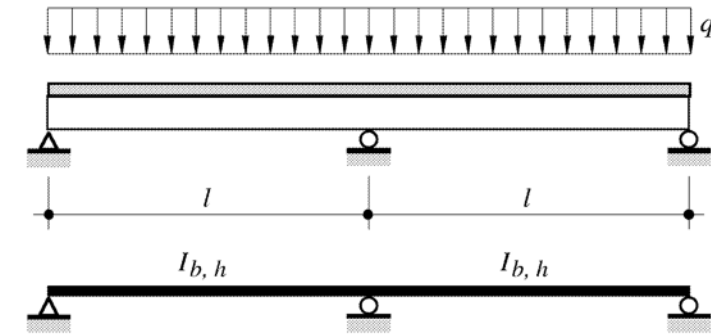
For continuous composite beams of constant height, the bending moments obtained by elastic analysis can be modified by reducing the maximum bending moments at supports by the percentages shown in Table 4 and increasing the corresponding span moments accordingly.

Table 4: Maximum reduction in bending moments resulting from elastic analysis

Section class in areas of negative bending moments	1	2	3	4
Elastic calculation without taking cracking into account	40%	30%	20%	10%
Elastic calculation including cracking	25%	15%	10%	0%

4.2.3.5 The ultimate plastic resistance of the cross-sections of continuous beams as well as of laterally held frames that meet the requirements of section class 2 may be used in the verification, even if the cross-sections do not meet these same requirements in the supports area, provided that the elastic resistance of the cross-section in supports area is only used to a maximum of 90% for the load case under study. The torsional buckling check in the supports area must then be carried out in accordance with SIA 263.

Determination of internal forces, M_{Ed} : Elastic calculation, redistribution



Value for e.g. class 2:

$$r = 30\%$$

$$M(x) = -\frac{q_{Ed}(L-x)^2}{2} - \frac{0.7q_{Ed}L^2}{8} + R_B(L-x)$$

$$x_{\max} = \frac{3.3L}{8} = 0.4125L$$

$$M_{\max}^+ = 0.122 q_{Ed}L^2$$

In relation to:

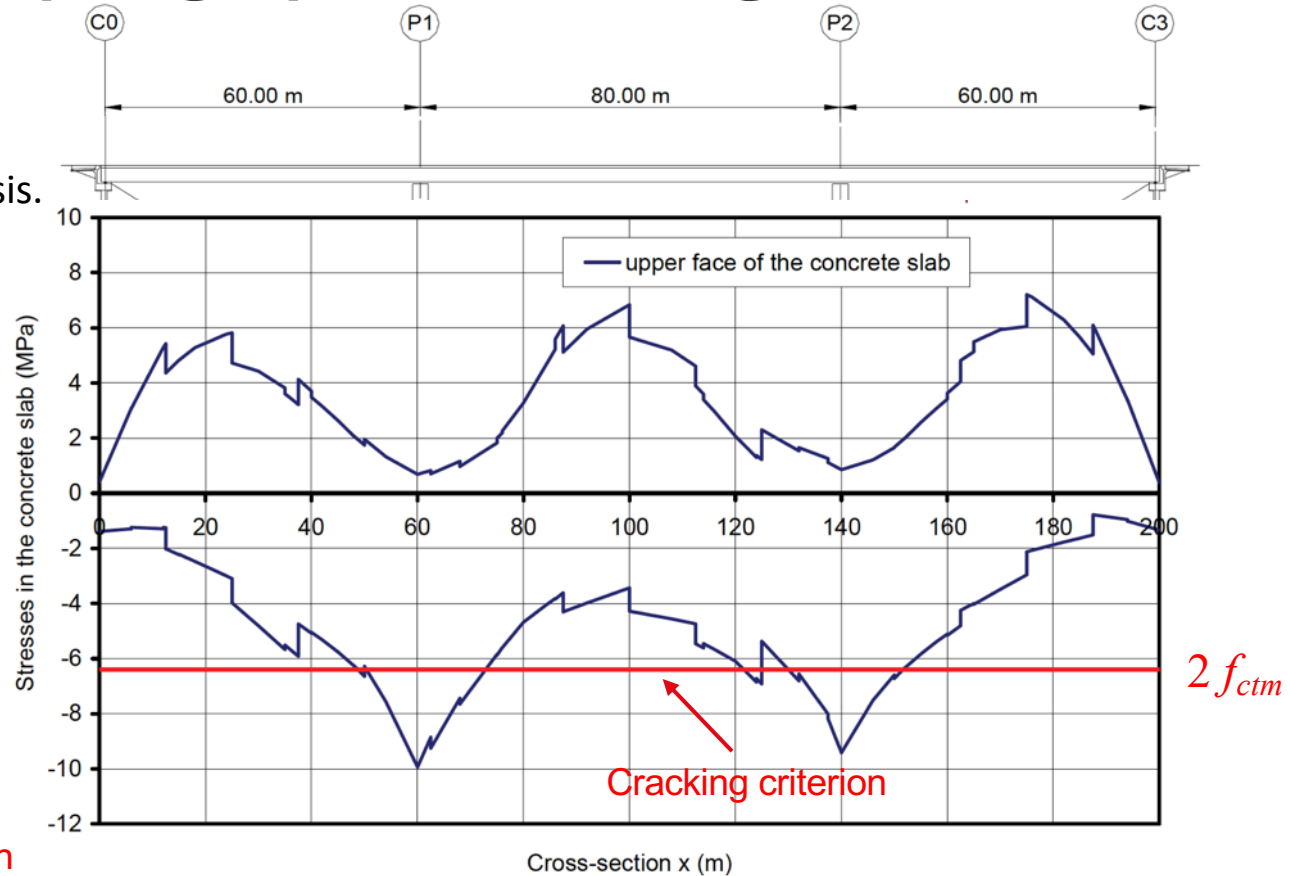
$$\text{Without redistribution, } M_{\max}^+(0.37L) = 0.0703 q_{Ed}L^2$$

Redistribution, without computing $x_{\max}$ (after redist.)

$$M_{\max}^+ = (0.0703 + 0.3/8)q_{Ed}L^2 = 0.108 q_{Ed}L^2$$

Example (bridges) for calculating cracked zones

1^{ère} iteration,
uncracked analysis.



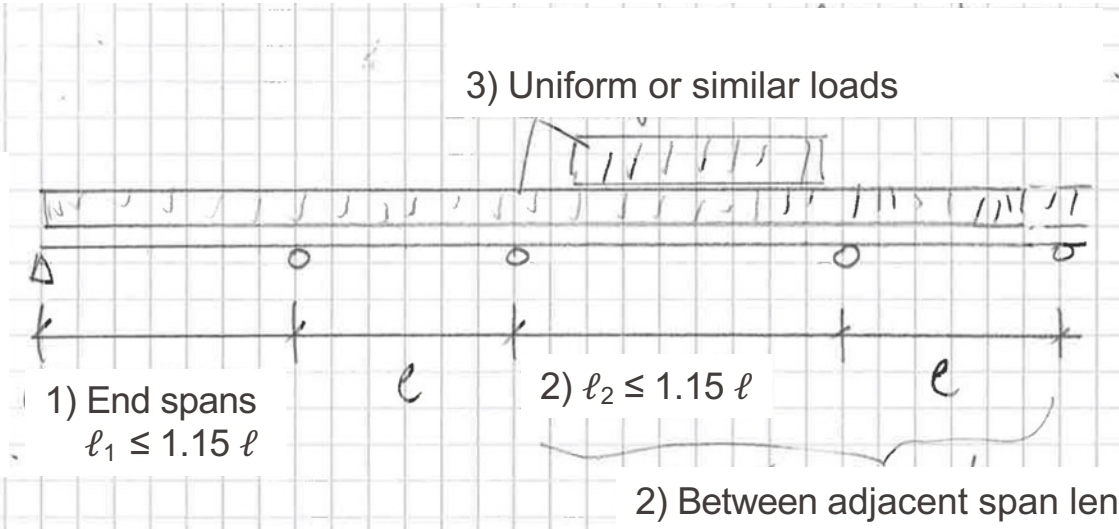
Determination of internal forces, , plastic calculation (PP)

For a plastic-plastic calculation (only for buildings):

- Section class 1, bisymmetrical steel cross-section
- Closer to real ultimate state behaviour
- 3 additional requirements:

SIA 264 § 4.2.3.1

Note 3) Otherwise risk of partial mechanism, which is not considered here



Structural safety

TGC 10 § 4.7.4

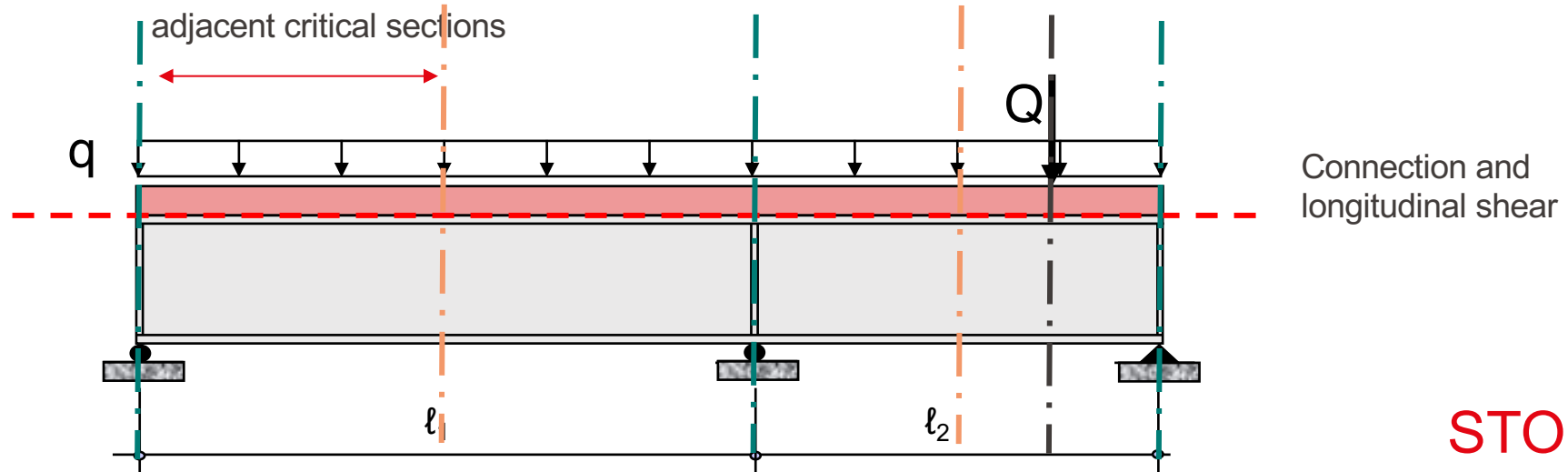
TGC 11 §10.5.3

Critical sections

- On supports (V , or $M^- + V$), including LTB
- In span, M_{max}^+
- At loads Q positions
- At major cross-section changes

Shear "length" :

Distance between 2 adjacent critical sections



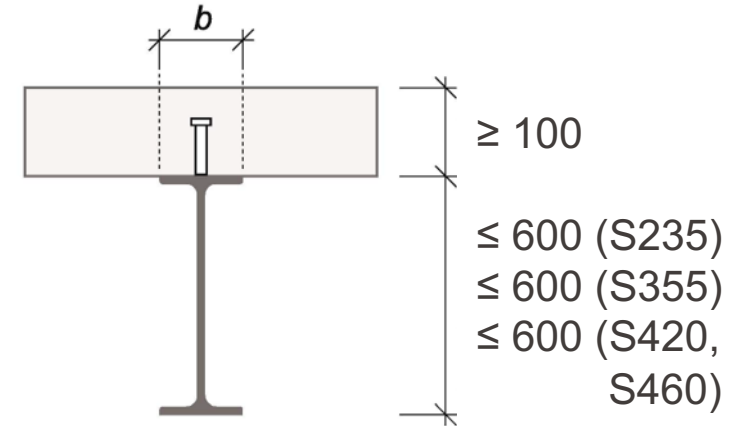
Not necessary to check for LTB at supports of continuous beams if meeting following maximum profile heights (buildings):

SIA 264 §5.1.2.6

- IPE up to 600 mm (S235), or 400 mm (S355), or 270 mm (S420,S460)
- HE up to 800 mm (S235), or 650 mm (S355), or 500 mm (S420,S460)

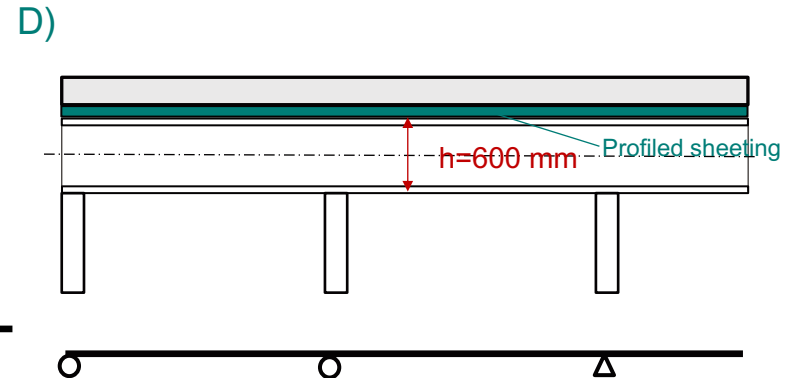
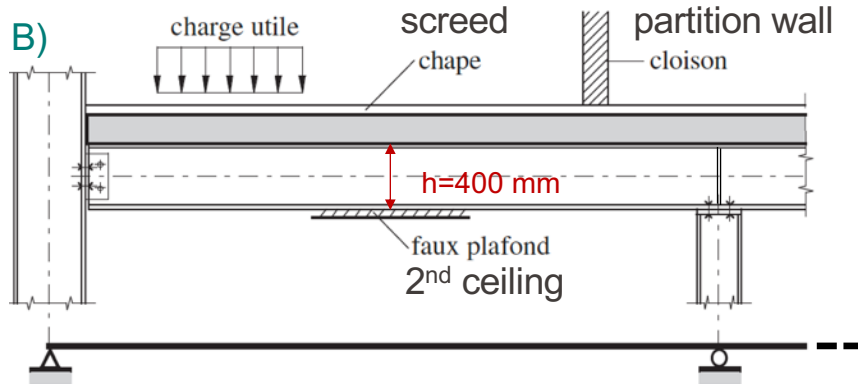
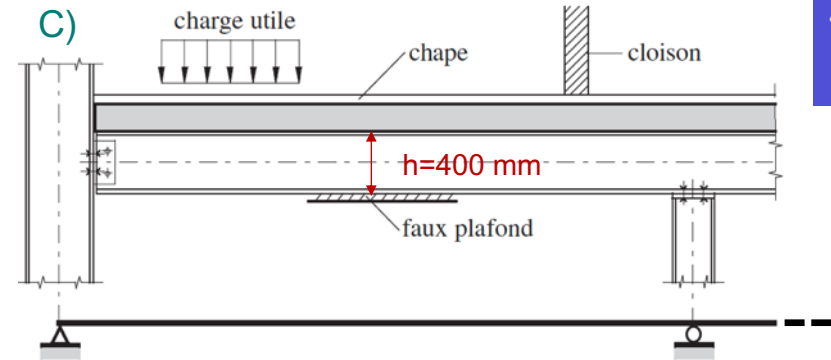
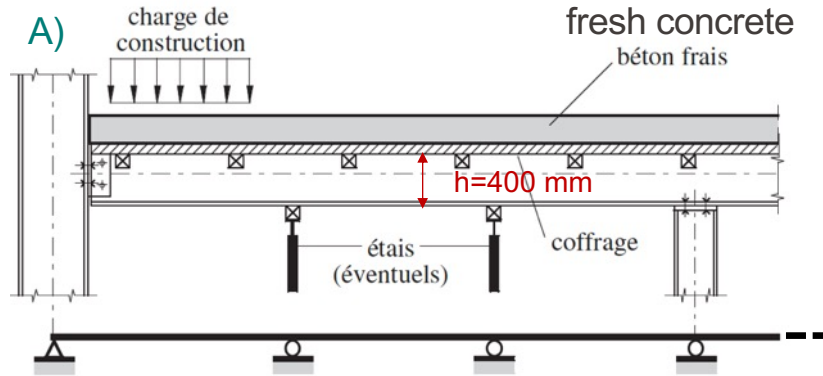
Prerequisites:

- Vertical stiffeners on supports
- Connection via headed studs
- Slab thickness min. 100 mm (prevents section rotation)



Checking on discharge

7) In which situation should IPE beams LTB be checked (several possibilities)?



Design/Sizing method, checks

- Whatever the class, **elastic checks** under M^+ (compressed concrete slab) or under M^- (tensioned and cracked concrete slab):
 - for all fibres in the section (structural steel, concrete, passive reinforcement)

$$\sigma_{Ed,a} = \frac{M_{el}}{I_b} z_b \qquad \sigma_{Ed,c} = \frac{1}{n_{el}} \frac{M_{el}}{I_b} (z_b - h)$$

The load cases are added together

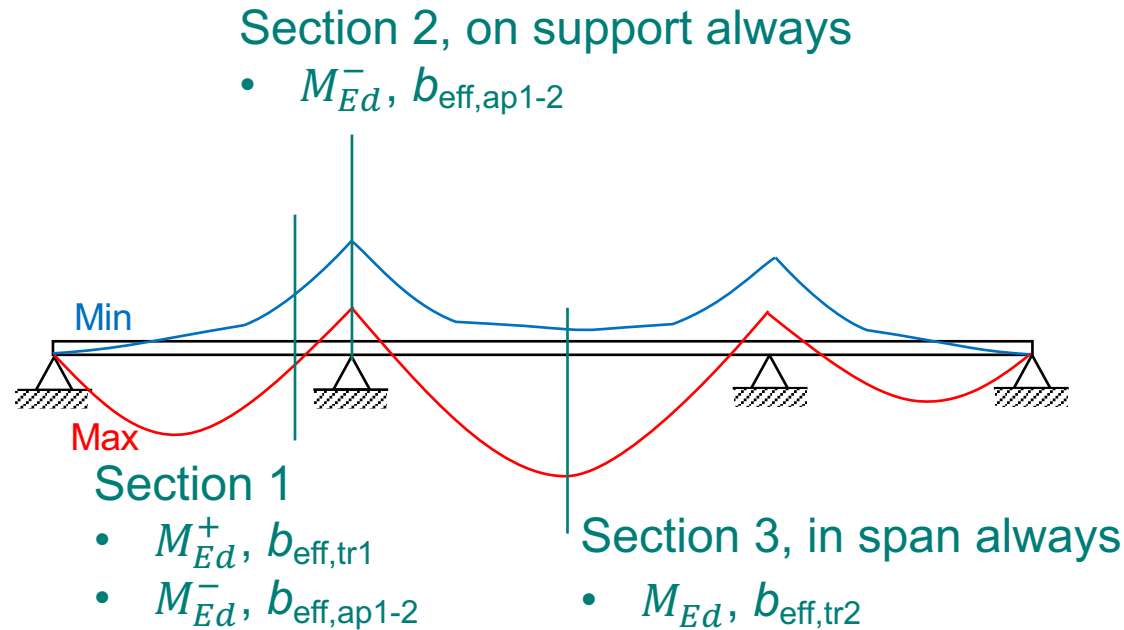
$$\sum_i \sigma_{Ed,i} \leq \frac{f_k}{\gamma_M}$$

- For class 1 or 2 sections, **plastic check** :

$$M_{Ed} \leq M_{Rd} = M_{pl,Rd}$$

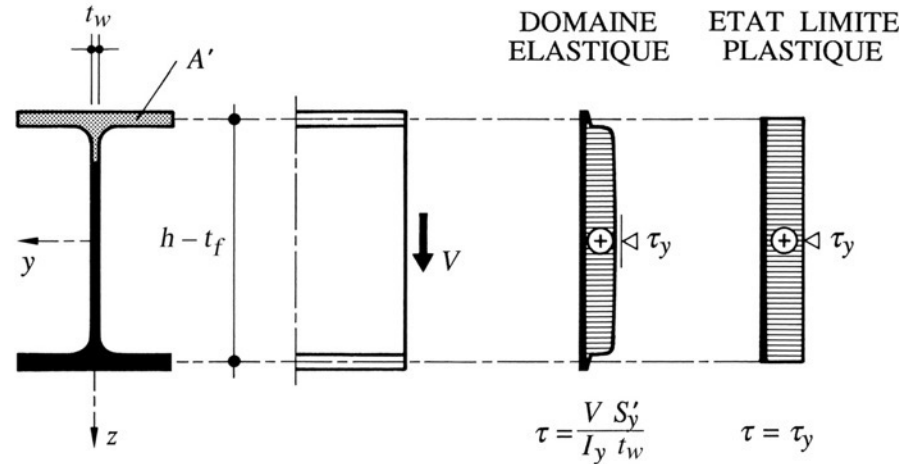
Section checks (EN)

Envelopes of moments & slab effective width



Shear strength (SIA 264 §4.3.2.2)

- Shear strength → same as steel beam → SIA 263
- Shear force taken by the steel section web
- M+V interaction, also in accordance with SIA 263



$$V_{Rd} = \frac{f_y \cdot A_v}{\sqrt{3} \cdot \gamma_{M1}}$$

Castellated profiles (SZS C5, p.84): approx. with (A_w - opening), or according to producer tables

Construction stage Loads to be considered

TGC 10 § 5.8.3

TGC 11 § 10.5.3 and 10.5.4

TGC 11, fig. 10.24: summary of different stages

Construction stage

Compressed upper flange, LTB at construction stage !

Final stage

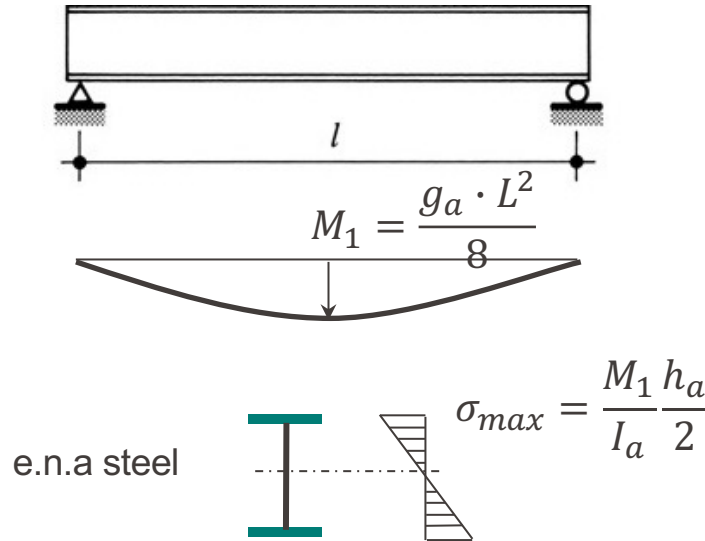
	Structure porteuse Système statique de la poutre	Section résistante	Charges et actions
Stade de construction			• Profilé (poids propre)
			• Profilé • Coffrage • Béton frais • Charge de construction
Stade définitif			• Profilé • Béton durci • Réactions des étais
			• Actions permanentes • Charge utile - longue durée - courte durée • Actions indirectes

Description of composite structure construction stages

■ STEP 1: Erection of the steel beam (using a crane, for example)

One looks at a beam subjected to sagging (positive bending).

One considers the most stressed section at mid-span.



Load to be carried :

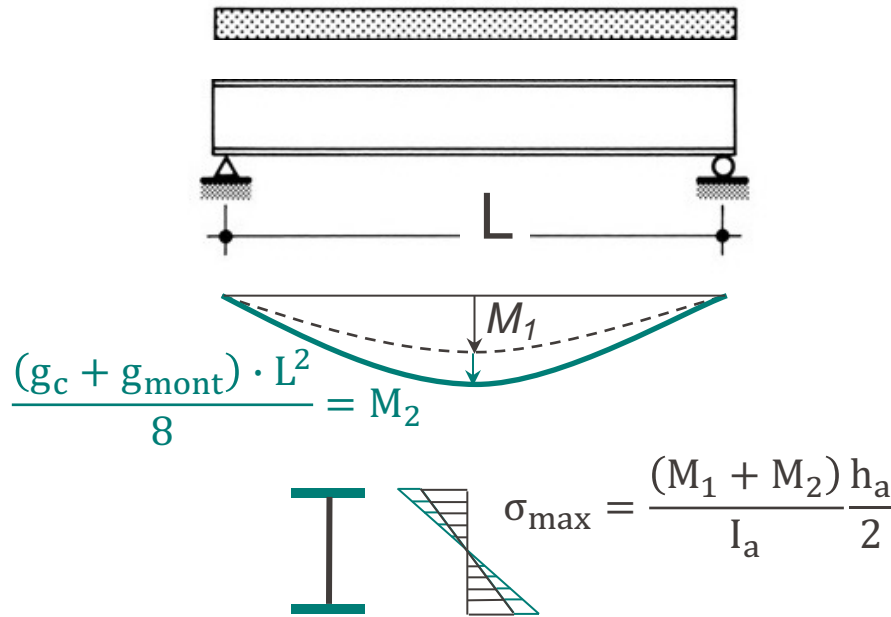
- steel profile self-weight g_a (N/m)

Resistant section :

- steel section A_a
- inertia I_a

Description of composite structure construction stages

STEP 2: Concreting the concrete slab



Load to be carried :

- Self-weight of timber formwork, concrete reinforcement and fresh concrete g_c (N/m)

Resisting section :

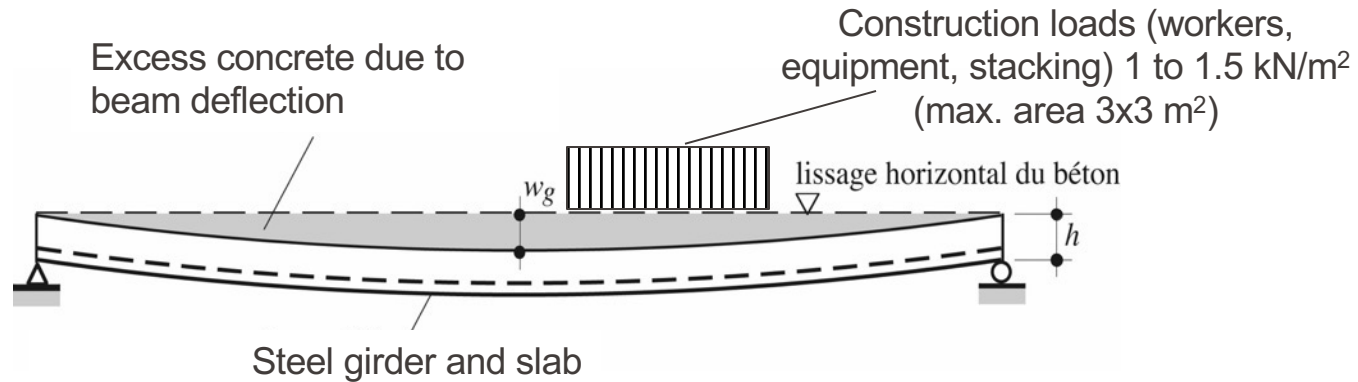
- steel only section A_a
- inertia I_a

Transitional situation where the steel section is subjected to the greatest stress.
EE calculation

Watch out for LTB !!!

TGC 11, fig. 6.5: beware of deformation during this stage!

STEP 2: Concreting of concrete slab



In addition of stress checks:

SIA 264 § 5.4.4.4

- Compute deflection, surplus to be considered if : $w_g \geq h/10$
- Deflection limit (w/o surplus): $w(g_a + g_c + (g_{\text{surplus}}) + g_{\text{montage}}) \leq L/300$
- Differences of M , w between parabolic and uniform load?

$$M(L/2)_{\text{para}} = 5g_{\text{surplus}} L^2/48$$

5/6

$$w_{\text{max,para}} = 61g_{\text{surplus}} L^4/(5760EI)$$

61/75

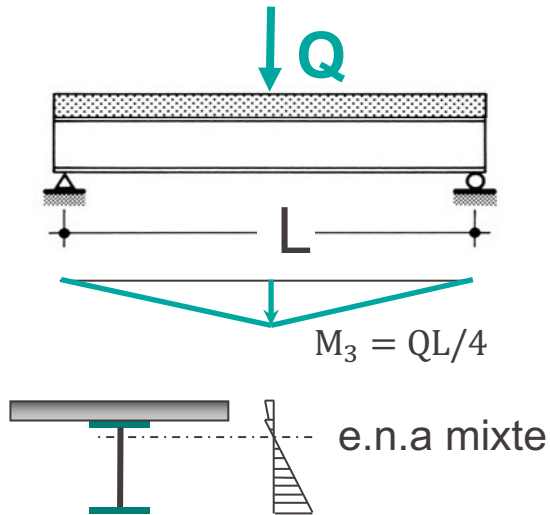
$$\leftrightarrow g_{\text{surplus}} \approx 0.7w_g\rho_c$$

Description final stage composite structure

- STEP 3: The concrete hardens and becomes resistant

The behaviour of the structure becomes **composite**

- STEP 4: Application of other permanent and live loads



Load to be carried :

- e.g. concentrated load Q (N)

Resisting section: steel + compressed concrete

- section $A_b = A_a + (A_s) + A_c / n_{el}$
- inertia I_b

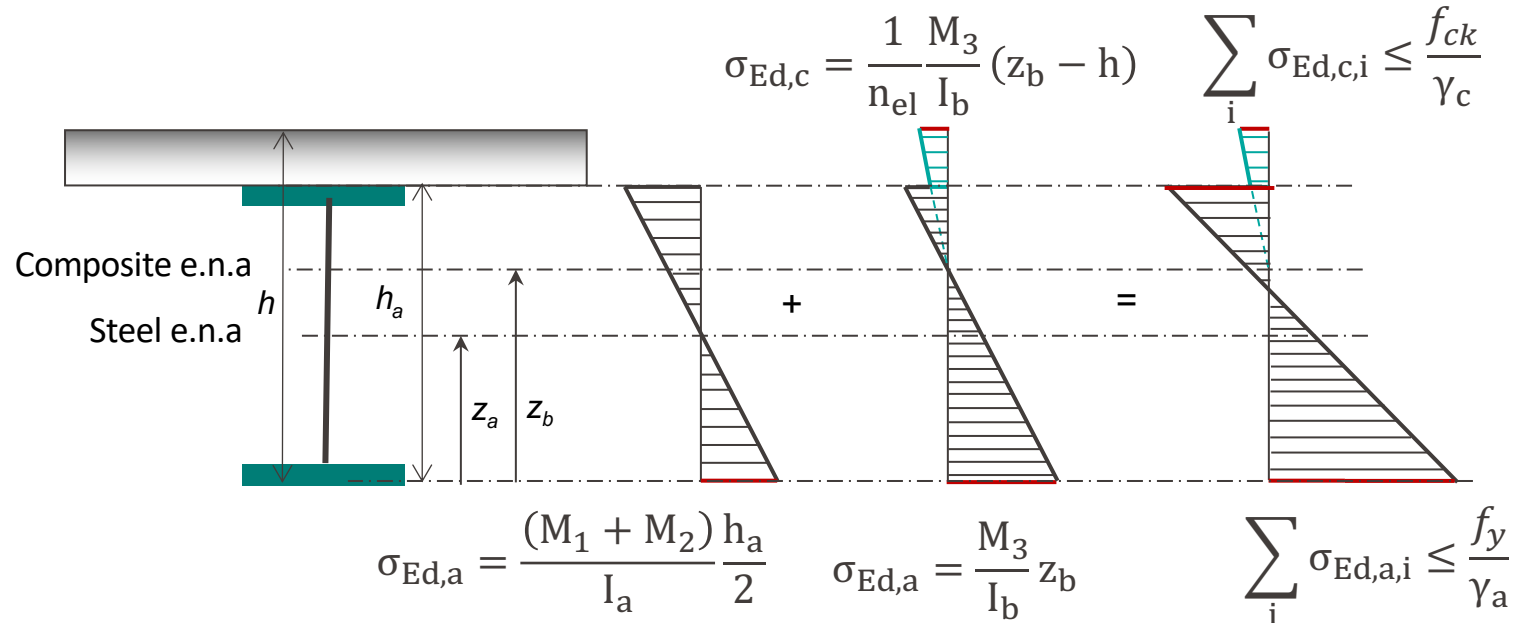
For the calculation of deflections and for the EE calculation:
Value of the coefficient n_{el} of equivalence to be defined

Addition of construction and final stages

- Addition at the **final elastic stage**

$M_{Ed,a}$ = moment taken by the beam in steel-only behaviour

$M_{Ed,b}$ = moment taken by the beam in composite behaviour



Note: several diagrams depending on the duration of load application (fct of $n_{el,i}$)

Addition of construction and final stages

- Addition at the **final elastic stage**

$M_{Ed,a}$ = moment taken up by the beam in steel-only behaviour

$M_{Ed,b}$ = moment taken up by the beam in composite behaviour

ELU, steel stress:

$$\sigma_{Ed,a} = \frac{M_{Ed,a,1-2}}{I_a} z_a + \sum_i \frac{M_{Ed,b,i}}{I_{b,i}} z_{b,i}$$

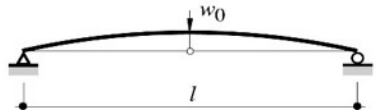
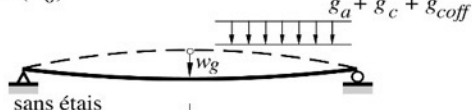
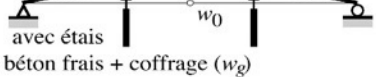
ELU, concrete stress:

$$\sigma_{Ed,c} = \sum_i \frac{1}{n_{el,i}} \frac{M_{Ed,b,i}}{I_{b,i}} (z_{b,i} - h)$$

Serviceability

TGC 11 § 10.5.9

TGC 11, fig. 10.46: Checking deflection limits (elastic)

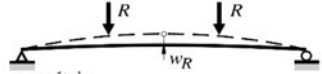
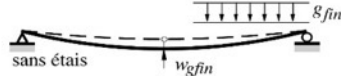
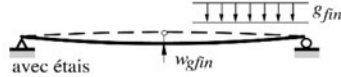
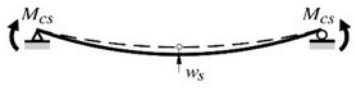
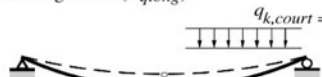
	Système statique Charges et actions (flèche w)	Section résistante	Exigences Flèches	Vérifications
Stade de construction	 Contreflèche (w_0)	 El_a	Aspect	
	 sans étais	 El_a	$w_0 \geq \sqrt{0.025} l \geq 10 \text{ mm}$ $w_g = w_1 + w_{31}$	$w_0 + w_1 + w_{31} \leq \frac{l}{300}$
	 avec étais Poutrelle + béton frais + coffrage (w_g)			

Indices (according to TGC):

0 = precambering
 1 = self-weight
 2 = other self-weights (and shrinkage defl.)

3 = live loads
 30 = rare (= short duration)
 31 = frequent (= short duration)
 32 = quasi-permanent (= long term)

TGC 11, fig. 10.46: Checking deflection limits (elastic)

Stade définitif	 avec étais Réactions des étais (w_R)	 $El_b (n \varphi)$	Aptitude au fonctionnement $w_R = (w_1 + w_2)'$ $w_{gfin} = (w_1 + w_2)''$ $w_{qcourt} = w_{31}$
	 sans étais w_{gfin}	 $El_b (n \varphi)$	éléments ductiles $w_0 + w_1 + w_2 + w_{31} \leq \frac{l}{350}$
	 avec étais w_{gfin}		éléments fragiles $w_0 + w_1 + w_2 + w_{30} \leq \frac{l}{500}$
	 Retrait w_s	 $El(n_s)$	Aspect $w_s = w_2''$ $w_{qlong} = w_{32}$
	 Charge utile à long terme (w_{qlong}) $q_{k, long} = \psi_2 q_k$	 $El_b (n_t)$	$w_0 + w_1 + w_2 + w_{32} \leq \frac{l}{300}$
	 Charge utile à court terme (w_{qcourt}) $q_{k, court} = q_k \text{ ou } \psi_1 q_k$	 $El_b (n_0)$	Confort $w_{31} \leq \frac{l}{350}$

$$w_2 = w_2' + w_2'' + w_2'''$$

Addition of construction and final stages

- Addition to the **final elastic stage**

$M_{Ed,a}$ = moment taken by the beam in steel-only behaviour

$M_{Ed,b}$ = moment taken up by the beam in composite behaviour

ELU, steel stress:

$$\sigma_{Ed,a} = \frac{M_{Ed,a,1-2}}{I_a} z_a + \sum_i \frac{M_{Ed,b,i}}{I_{b,i}} z_{b,i}$$

ELS, deflection:

$$w_{tot} = \frac{\alpha_i \cdot q_{Cd,a,1-2}}{E_a I_a} + \sum_i \frac{\alpha_i \cdot q_{Cd,b,i}}{E_a I_{b,i}}$$

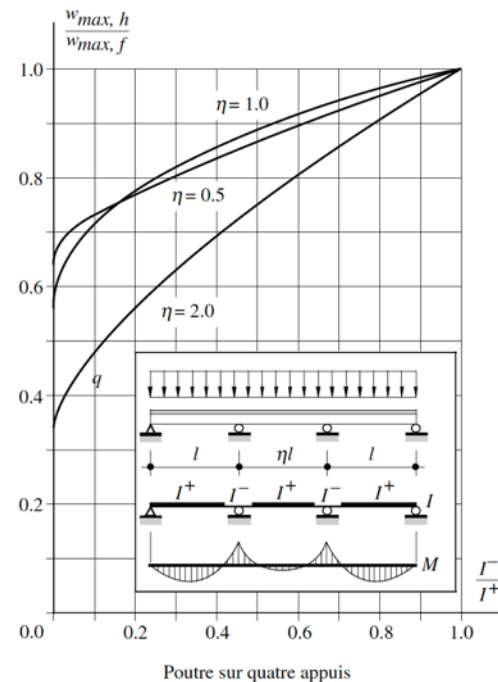
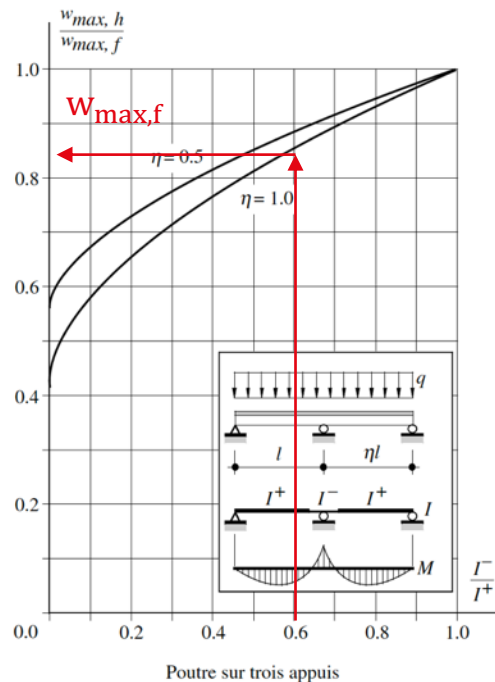
α_i coefficient fct static system & loading $(\frac{L^3}{48}, \frac{L^4}{384}, \frac{5L^4}{384}, \dots)$

Checking deflections, influence of concrete cracking (TGC11, fig.10.51)

SIA 264 § 4.6.4, § 5.1.3:

- Internal forces and deflections at ELS
- Elastic calculation method
- Take into account cracking, creep and shrinkage of concrete.

In addition, take into account the flexibility of connection, if significant (e.g. partial connection).



Special loads to consider

- Prop(s)
- Concrete shrinkage

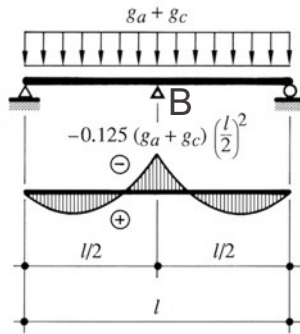
TGC 10 § 5.8.3 and 5.8.5

TGC 11 § 10.5.5

TGC 10, fig. 5.37: mixed structure construction with/without prop

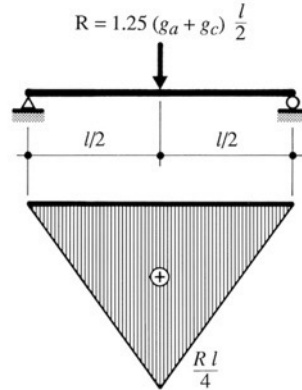
Action-reaction principle

(a) Supported beam



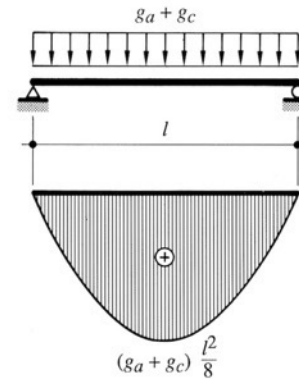
Resisting section
= steel only

(b) Removed prop



Resisting section
= composite

(c) Final stage



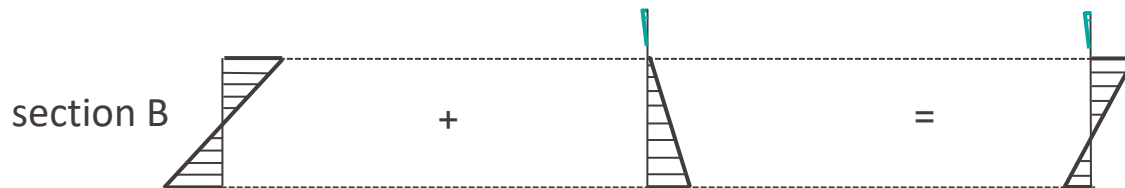
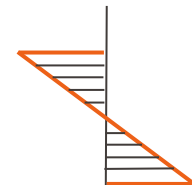
Resisting section
= composite

In the end, in both cases:

$$\sum F_i = 0$$

$$\sum F_i \cdot z_i = M_B$$

Without prop



Composite structure construction with/without prop

In the case of a supported beam

- Reduced deflection during concreting
- In the mid-span section, the steel beam is subject to less stress due to the transfer of part of the self-weight of the fresh concrete to the composite beam. For example :

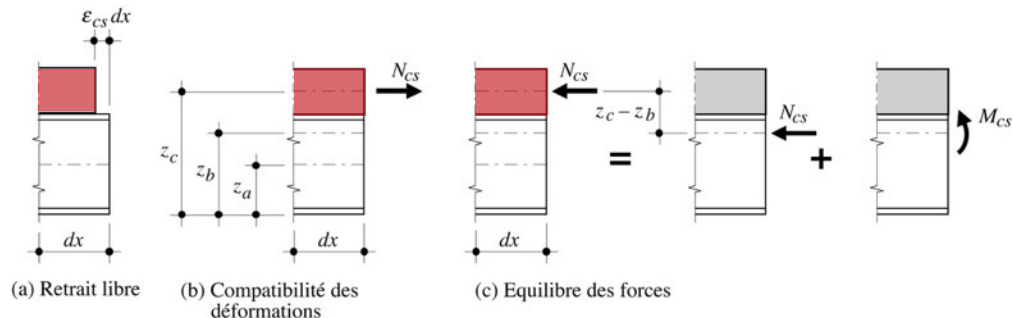
$$M_{a,Ed} = M_1 + M_2 = \frac{0.5(g_a + g_c)L^2}{4} \quad \text{without strut}$$

$$M_{a,Ed} = - \frac{0.125(g_a + g_c)L^2}{4} \quad \text{with prop}$$

Shrinkage phenomenon and internal forces

- Shrinkage = long-lasting action
- SIA 262: $\varepsilon_{cs}(\infty) = 0.5\text{‰}$

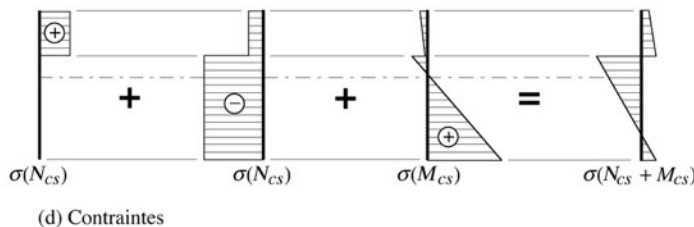
$$N_{cs,Ed} = \gamma_G \cdot \varepsilon_{cs,\infty} \cdot E_{cs} \cdot A_c \qquad M_{cs,Ed} = N_{cs,Ed} \cdot \left(h - \frac{h_c}{2} - z_b\right)$$



Self-balancing stress state

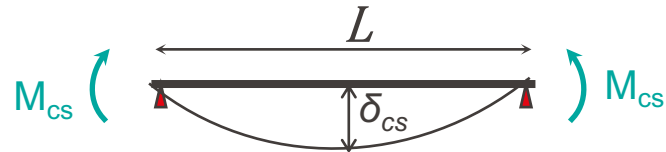
Determinate systems
 $M_{Ed}(\text{shrinkage}) = 0$

But vertical deformations



Calculation of deflections and moments in a composite beam on 3 supports

- The internal moment due to shrinkage M_{cs} creates a deflection (important for verifying serviceability).

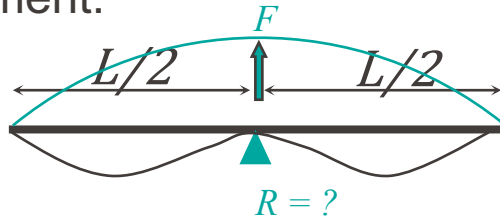


$$\delta_{cs} = \frac{M_{cs} \cdot L^2}{8E_a I_{mixte}}$$

- Case of a hyperstatic beam, beam on 3 supports :

TGC11 Fig. 10.49

The intermediate support prevents the formation of the deflection expected if the beam were isostatic. Creation of a hyperstatic shrinkage moment.



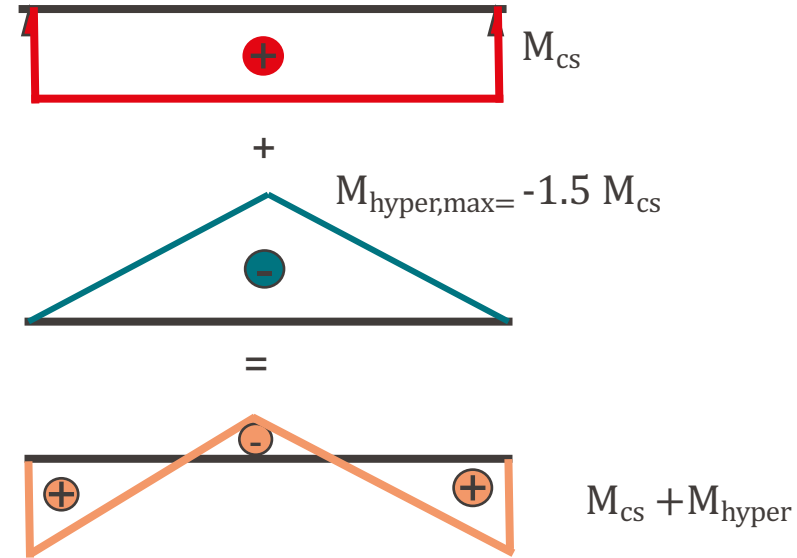
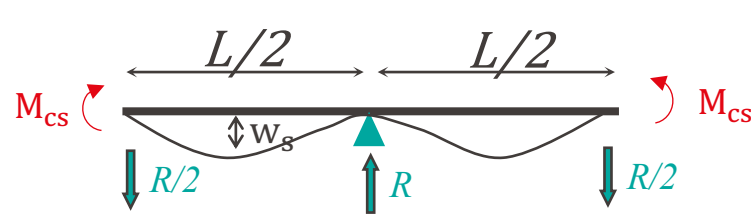
$$\delta_{F, \text{midspan}} = -\frac{F \cdot L^3}{48E_a I_{mixte}} = -\delta_{cs}$$

$$R = \frac{6 \cdot M_{cs}}{L}$$

Permanent support (not a prop)

Calculating deflections and moments in a composite beam

- Case of a indeterminate beam, beam on 3 supports :



- Deflections: (TGC11, eq.10.104 to 10.106)

pour une poutre simple: $w_s = \frac{M_{cs} l^2}{8 E_a I_{b,h}}$

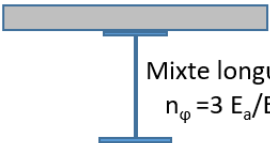
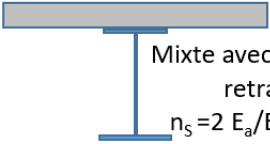
pour une poutre sur trois appuis: $w_s \cong \frac{M_{cs} l^2}{27 E_a I_{b,h}}$

pour la travée de rive d'une poutre sur quatre appuis et plus:

$$w_s \cong \frac{M_{cs} l^2}{20 E_a I_{b,h}}$$

Appendices

Recall: resisting section $\Leftrightarrow$ action

Modèle équivalent	Section résistante	Actions
Modèle 0 	Acier seul 	-poids propre de la poutre métallique -poids propre du béton frais -charge de montage
Modèle 1 	Mixte longue durée $n_{\varphi} \approx 3 E_a / E_{cm} = 18$ 	-Revêtement -Glissière -Fluage
Modèle 2 	Mixte avec effet du retrait $n_5 = 2 E_a / E_{cm} = 12$ 	-Effort normal dû au retrait N_{cs} -Moment dû au retrait M_{cs}
Modèle 3 	Mixte courte durée $n_0 = E_a / E_{cm} = 6$ 	-Trafic (Q_k et q_k) -Température

Calculation of internal forces ACCORDING TO TGC 12, chap. 13

Inertia can change along the beam (variable steel section, variable b_{eff} , cracking on interm. supports).

Method 1:

- A single analysis: elastic calculation of forces, taking into account the participation of tensile concrete everywhere, with $I_{b,h,\text{cte}}$ (building) or $I_{b,h,\text{variable}} = f_{ct} (A_a, b)_{\text{eff}}$
- Flat-rate redistribution r % of intermediate supports to bays (depending on support cross-section class)
- Only for actions taking place after the link has been created.

Method 2a:

- 1^{ère} analysis: elastic calculation of forces taking into account the participation of concrete stretched everywhere
- Under ELS characteristic load (rare load case), including long-term effects
- In zones M^- , lengths L_{fiss} where, inertia of cracked sections
- 2^{ème} analysis: elastic stress calculation with I_{variable} and cracked sections.

Method 2b:

- In-situ concrete slab: analysis with $L_{\text{fiss}} = 15\%$, elastic calculation of forces $I_{b,I}$ and $I_{b,II}$ (limit $L_{\text{min}}/L_{\text{max}} > 0.6$).

Method 2a: internal stress calculation diagram

1^{ère} analysis

